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Accuracy of Approximation

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Abstract
It is introduced, here, and defined a measure of accuracy of approximation, of a function by another. Few examples are presented to illustrate its usage.

Keywords:- limit, convergence, accuracy, approximation, series, factorials, Sterling.

Introduction

From a practical point of view, limit is a tool for approximation. Theoreticians avoid the issue of efficiency, in any measure, of the approximation – It is usually required that " if n is large enough" or "if n is sufficiently large". An attempt is made to introduce a measure of accuracy of the

approximation that could be used as a criterion to determine the size of n as required and conversely that measure may be calculated for a given approximation. Also this same measure could be used as a criterion to choose an approximate when more than one choice is available for more details and wider background see [5, 6].

Definitions and Assumptions

[x] = The largest integer equals to or less

than x, or reference no. x.

$\{S_n(x)\}$, n = 1,2…. is a sequence of real valued functions converging

to $f(x) > 0$, $S_n(x) \geq 0$, [1]

$Vn(x) = \left|\dfrac{f(x) - Sn(x)}{f(x)}\right| < 1$ is the relative

error at n.

$d_n(x)=1-V_n(x)$ is the degree of the accuracy at n. (1)

Procedure

Problems of approximation are very diverse in their nature and purpose, hence it would be inadvisable to try to obtain an algorithm to solve these or some of these problems.

We confine this study to solve the following problem for few cases as examples.

Problem: For a given degree of accuracy d (x), what should be the value of n so that $S_n(x) \approx f(x)$ with deg of acc., d(x), and conversely?

As for another application, the problem is:

of two sequences $\{T_n\}$ and $\{S_n\}$ ∍ $\lim T_n = \lim S_n = f(.)$, which one is preferred to approximate f(.)?

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Applications

the Geometric series

A geometric series is a power series [2]:

$1 + u(x) + [u(x)]^2 + [u(x)]^3 + \ldots\ldots$ and for $|u(x)| < 1$ we have

$$\lim\ (1+u(x) + [u(x)]^2 + \ldots\ldots + [u(x)]^{n-1}) = \frac{1}{1-u(x)}$$, as n approaches infitity,

To simplify the notations we shall write u for u(x); thus

$$S_n = 1+u+u^2+\ldots+u^{n-1} = \frac{1-u^n}{1-u} \text{ and}$$

for

$|U| < 1$

$$\lim S_n = \frac{1}{1-u}$$

Our purpose in this section is expressed by

Assertion 1

Let d denote the deg. of acc. Then for $0 < u < 1$

$$S_n \approx \frac{1}{1-u} \text{ iff } n = [\ \frac{(\ln(1-d)}{\ln u}] + 1$$

Proof

We have $d = 1-u^n$ since

$$\frac{S_n}{\lim S_n} = \frac{1-u^n}{1-u}(1-u) < 1 \quad \text{(see 1)}$$

The proof is obtained by solving for n

$$u^n = 1-d \Rightarrow n = \frac{\ln(1-d)}{\ln u}$$

The proof of the converse is left for the reader.

Example : For $u = \frac{1}{2}$ and $d = 0.99$ we find $n = 7$

Conversely for n=5, $u = \frac{1}{2}$ we find d=0.967

The Exponential e^x

Our aim in this section is to determine the value on n which corresponds to a prearranged deg. Of acc., d , so that [1]

$$e^x \approx s_n(x) = 1+x+x^2/2! + \ldots + x^n/n! \quad (2)$$

To justify the above inequality we notice that every pair of corresponding terms at the two infinite series satisfies the inequality so do the corresponding partial sums. It is easy to prove that limits of the partial sums do too.

Alternating Series

An alternating series is defined to be

$S = \lim S_n$, where

$S_n = u_1 - u_2 + u_3 - u_4 + \ldots \pm u_n)$

and $u_n > 0$. We assume

$u_n > u_{n+1}$, n= 1,2,...

$\lim u_n = 0$ (3)

So that the series is convergent .For details, the reader is referred to [3]. Since $\lim S_{2m} = \lim S_{2m-1} = S$. we shall consider n=2m m=1,2,...

The aim in this section is to determine the value of n so that $S \approx S_n$ with a preassingned deg. of acc.,= d.Since for m= 1,2,..., $S_{2m} < S < S_{2m-}$

we have, according to (1)

$$d = \frac{S_{2m}}{S} \qquad (4)$$

Now let for k= 1,2,…,2_{m-1}
$W_k = u_k - u_{k+1}$
Then their arithmetic mean is

W=1/ $2_{m-1}(u_1-u_2 + u_2-u_3 + u_{2m-1} - u_{2m}) = (u_1-u_{2m}) / 2_{m-1}$
and $S_{2m} = (w_1+w_3+\ldots+w_{2m-1})$
It simplifies matter and for convenience we represent S_{2m} by a very well statistically acceptable representation, namely.

$$S_{2m} \approx m\bar{w} = \frac{m(u_1 - u_{2m})}{2m-1} \qquad (5)$$

From (4) and (5) we have
S_{2m} = Sd and S< u_1,

$$\frac{m}{2m-1}(u_1 - u_{2m}) = Sd \le u_1 d$$

From (3), m (u_1-u_2) < m (u_1-u_{2m})
< (2m-1) u_1d (6)
which yields

$$m > \frac{u_1 d}{(2d-1)u_1 + u_2}$$

Thus we have just proved

Assertion 2

If $n = 2m > \dfrac{2u_1 d}{(2d-1)u_1 + u_2}$ then

S ≈ S_n with deg. of accuracy = d. Conversely if
S ≈ S_{2m} then the deg. of ace. is given by

$$d > \frac{m(u_1 - u_m)}{(2m-1)u_1} > \frac{m(u_1 - u_2)}{(2m-1)u_1}$$

Remark: In practice, the value of n would be

$$n_0 \ge \left[\frac{2u_1 d}{(2d-1)u_1 + u_2}\right] + 1$$

Factorials and Sterling Formula

For a positive integer n, n factorial is defined to be n! = 1.2.3. ….n
while Sterling formula is an asymptotic formula that expresses n! as [4]

$$n! = \left(\frac{n}{e}\right)^n \sqrt{2\pi n}\, e^{r(n)} \qquad (7)$$

where

$$\frac{1}{12n+1} < r(n) < \frac{1}{12n}$$

our purpose in this, section is to justify the following:

Assertion 3

$n! \approx (\frac{n}{e})^n \sqrt{2\pi n}$ = A(n) with dog. Of

acc. = d iff. $-\dfrac{1}{12n} < \dfrac{1}{12n+1}$

Proof

Since $e^{r(n)} > 1$, A(n) < n!
hence, according to (1)
$d = e^{-r(n)}$, therefore ln d = -r(n)
As an example, for n = 3

$6 \approx (\frac{3}{e})\ \sqrt[3]{2\pi 3}$ with acc. = d unless

0.973 < d < 0.974

Conclusion

A measure called "degree of accuracy" has been introduced that can be used as a criterion to determine the closeness of a function to another when one of them is approximating the other.

This measure is given in (1). Also it may be used to compute the degree of accuracy when the two functions depend on, n, as is shown in the above example.

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لەم توێژینەوەیە دا پێوانەی ووردی کراوە بۆ نزیك کردنەوەی نەخشیەك بە هۆی یەکێکی تر. هەندێ نمونە نیشان دراوە بۆ روون کردنەوەی ئەو بەکار هێنانە.

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الخلاصة

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